Vertical profile of turbidity and coda Q

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SUMMARY

The well-known decrease of scattering capability of the Earth medium with increasing depth is parametrized as a power-law decay of turbidity, or scattering coefficient. The single isotropic scattering approximation for body waves including scattering loss is derived for this profile. An asymptotic case is analysed where the upper constant-turbidity layer is thin, i.e. coda lapse time is much larger than the traveltime across this layer. In this important case, power-law coda shapes are obtained. Predictions of this model are tested against relatively abundant data on coda Q and its frequency dependence, and also against some coda shapes. For filtered or spectrally analysed data, agreement is reasonable, explaining in particular the lapse-time dependence of coda Q, and, to a lesser extent, the exponent of the Q versus frequency power law. The data can be fit by turbidity decay as the inverse square or cube of depth and intrinsic Q of the order of 2000.

Key words: scattering, seismic coda.

INTRODUCTION

When Aki & Chouet (1975) proposed representing the observed coda waves of near earthquakes by a single isotropic scattering model (SISM), they ascribed the difference between model and real coda shapes to attenuation of some kind, expressed formally through an exponential decay multiplier containing a quality factor (Q), known as 'coda Q', or Q_c. Soon after, Rautian & Khalturin (1978) found that real coda shapes in a wide lapse-time range could not be described by a single Q_c value. Instead, real coda shapes show a step-like (Rautian & Khalturin 1978) or a gradual (Akamatsu 1980) increase of coda Q with lapse time.

Aki (1980) proposed interpreting both coda Q ($=Q_c$) and S-wave Q ($=Q_S$) primarily in terms of scattering loss. Dainty (1981) proposed a similar idea with regard to Q_S, and, assuming intrinsic Q to be frequency-independent and of the order of 1500–2500, came to the similar conclusion that, at frequencies of around 1–2 Hz, the scattering contribution to total Q is dominant. However, with the advent of more accurate multiple-scattering coda models, numerical (Gusev & Abubakirov 1987) and analytical (Shang & Gao 1988; Zeng, Su & Aki 1991), it became clear that, if a uniformly scattering medium is a reasonable model for the real Earth, the notion of scattering-controlled coda Q cannot be correct. Indeed, in such models coda decay does not have any exponentially behaving factor, so interpretation of data based on these models inevitably ascribes additional coda decay to intrinsic loss. As we try to show here, however, a scattering origin of coda Q is quite reasonable for non-uniform turbidity distributions.

Therefore, in the present study we reject the assumption of a uniform scatterer distribution and assume instead fast turbidity decay with depth. In general terms, fast turbidity decay with depth is evident from array observations of short-period teleseismic waves, which reveal a near-surface scattering ('turbid') layer covering a 'transparent' mantle (see, for example, Aki 1973; Flatte & Wu 1988). The first and seemingly unique attempt to determine vertical turbidity structure based on coda data was made by Rautian *et al.* (1981), who deduced from their data a rather steep overall depth decay.

In the following, a simple analytic model (SISM) is derived for a half-space with a power-law turbidity profile. Analytical results will then be compared with published observational data.

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THEORY

The approach of Aki & Chouet (1975), with some modifications, will be followed. Assume a source and a receiver to be collocated on the surface of a half-space with constant (body) wave velocity c and turbidity $g(h)$, where h is the depth coordinate in the half-space. Initially, no intrinsic loss is assumed. Let the source radiate at time $t=0$ a short energy pulse of energy equal to E in a frequency band around a central frequency f. At time t ('lapse' time), one can write the single-scattered wave energy which returns to the receiver from an element $d\Omega$ of solid angle Ω as a product:

$$dW(t)=\left[\frac{E\exp(-P)}{4\pi r^2}\right][g(h(r,\Omega))\,dr]\,(r^2\,d\Omega)\left[\frac{\exp(-P)}{4\pi r^2}\right], \tag{1}$$

where $r=ct/2$ is the distance to the spherical shell of 'active scatterers' (generating coda just at the time t), and

$$P=\int_o^r g(h(r_1,\Omega))\,dr_1 \tag{2}$$

is the one-way scattering loss, measured in terms of ln (power), on the way from $r_1=0$ to $r_1=r$ or back. On the right-hand side of eq. (1), the first factor is the direct wave energy density over a unit area of the shell; the second one represents the relative fraction of direct wave energy that will be scattered during wave propagation over a distance dr; the third one converts scattered energy on a unit area to the same energy in a solid angle $d\Omega$; and the last factor describes energy spreading and scattering loss on the wave's way back. The wave intensity, $I(t)\equiv dW(t)/dt$, produced by the entire shell is

$$I(t(r))=\frac{cE}{32\pi^2 r^2}\int_{\Omega_0} g(h(r,\Omega))\exp(-2P)\,d\Omega\,, \tag{3}$$

where Ω_0 is the lower hemisphere. Introducing the usual spherical coordinate system (r, ϕ, Θ), and integrating over azimuth ϕ, the basic formula

$$I(t)=I(t(r))=\frac{cE}{16\pi r^2}\int_0^{\pi/2} g(h(r,\Theta))$$

$$\times\exp[-2\int_0^r g(h(r_1,\Theta))dr_1]\,\sin\Theta\,d\Theta \tag{4}$$

is obtained. This result is valid for an arbitrary vertical turbidity profile. Now assume this profile to be the truncated power law

$$g(h)=\begin{cases} G, & 0<h<H,\\ G\left(\dfrac{H}{h}\right)^n, & h>H,\end{cases} \tag{5}$$

and substitute it into eq. (4). With the additional assumption that $H\ll r$ (or $t\gg H/c$), the integral can be evaluated for integer $n\geq 2$ as

$$I_n(t)=\left(\frac{cE}{16\pi}\right)C_n\left(\frac{ct}{2}\right)^{-n-2}, \tag{6}$$

where, for $n=2$, 3 and 4

$$C_2=\left(\frac{1}{4}\right)H\exp(-4GH);$$

$$C_3=\left(\frac{1}{9G}\right)H(3GH+1)\exp(-3GH);$$

$$C_4=\left(\frac{3}{512G^2}\right)H(64G^2H^2+48GH+18)\exp(-8GH/3). \tag{7}$$

The truncation (eq. 5) of turbidity values at small h takes place naturally for h so small that it is comparable to the wavelength (or else one can choose a value of $h=H$ to parametrize some real quasi-uniformly scattering surface layer). In any case, the problem of a singularity at $h=0$ is removed. Also, the assumption that $r\ll H$ is physically acceptable when we are interested in a later part of the record or coda. At $n=0$ (uniform half-space), the result is trivial:

$$I_0(t)=\frac{cEG\exp(-2Gr)}{16\pi r^2}=\frac{EG\exp(-GCt)}{4\pi c^2t^2}. \tag{8}$$

If we neglect here the scattering loss, and replace the exponent factor by unity (Born approximation), we obtain half of Aki & Chouet's (1975) result for a whole space:

$$I_{0B}(t)=\frac{cEG}{16\pi r^2}=\frac{EG}{4\pi ct^2}. \tag{9}$$

This function will be used as a normalizing/reference relation to determine theoretical Q_c.

Generally, for integer $n\geq 2$ and $r\gg H$, the Born approximation ($P=0$, eq. 2) yields

$$I_{nB}(t)=\frac{cE}{16\pi}\frac{nGH}{n-1}\left(\frac{ct}{2}\right)^{-3}. \tag{10}$$

'Interpolating' eqs (6) and (10) for non-integer $n>1$, one can expect that, in a medium with a (truncated) power-law turbidity profile (eq. 5), SISM will give $I(t)=I_n(t)=A_nt^{-2-n}$ if one accounts for scattering loss and $I(t)=I_{nB}(t)=B_nt^{-3}$ if one neglects it. Note that in the case of uniform turbidity, the Born approximation happens to be much better than SISM with scattering loss, because, as Aki & Chouet (1975) correctly noted, loss by single-scattered waves is compensated by contributions from multiply scattered waves (Gusev & Abubakirov 1987; Shang & Gao 1989). In contrast to this fact, ignoring scattering loss seems to be unacceptable in the case of fast vertical turbidity decay. This is not easy to prove with any rigour, however, and it will be checked numerically later. In the following, we will depart from eq. (6) and ignore eq. (10) as probably incorrect.

Now we can find the Q_c versus t relation for our model. Based on the definition of Q, and with eq. (9) as the reference relation, we obtain

$$Q=Q_n=2\pi f\left\{\frac{d}{dt}\ln\left[\frac{I_n(t)}{I_{OB}(t)}\right]\right\}^{-1}. \tag{11}$$

This gives the SISM coda Q value for a loss-less medium with a vertical power-law decay of turbidity:

$$Q_n=Q_c=\frac{2\pi ft}{n}. \tag{12}$$

Therefore, the model Q_c increases linearly with lapse time as well as with frequency. If a genuine intrinsic loss is also present in the medium, for the total $Q_c^{-1}=Q_t^{-1}$ we have

$$Q_t^{-1}=Q_n^{-1}+Q_{int}^{-1}. \tag{13}$$

In terms of coda shape proper, accounting for intrinsic loss modifies eq. (6) to

$$I_n^*(t)=\left(\frac{cE}{16\pi}\right)C_n\left(\frac{ct}{2}\right)^{-n-2}\exp(-2\pi ft/Q_{int}). \tag{14}$$

COMPARISON WITH OBSERVATIONS

To check the above theory against observations, published data were compiled on observed Q_c values, coda shapes, and the exponent β of the commonly assumed power-law frequency dependence of Q_c, where

$$Q_c = Q_0(f/f_0)^{\beta} \tag{15}$$

(typically, $f_0 = 1$ Hz). Q_c values and coda shapes were compiled for a single frequency band of 1–2 Hz, with central frequency 1.5 Hz; data for 1.25, 1.4 and 1.7 Hz were also used. If the data source gave Q_c results for 1, 1.25 and/or 2 Hz, they were interpolated or extrapolated based on eq. (15); for 3 Hz and larger they were extrapolated in the same manner but specially marked. It was sometimes not easy to relate the Q_c value to a particular lapse-time window. My choice, either based directly on the authors' statement or deduced from their data description, is given in Table 1. Errors, probably up to 1.5 times, are possible in some cases. The middle of a given interval was used as the lapse-time value in Figs 1–4.

Fig. 1 shows observed Q_c values for the most characteristic subset of studies, where Q_c was determined for more than for one lapse-time window, and by the most accurate techniques—using band filtering or direct spectral fit. For comparison, theoretical lines according to eq. (13) are given, for $n=2$ and 3, and $Q_{int} = \infty$ and 2000. The match is rather good, indicating that the model (eqs 5, 6, 7 and 13) is a reasonable one.

Fig. 2 shows Q_c results either (a) based on unfiltered records or (b) obtained with the band filtering or the spectral technique in a single, sometimes loosely defined, lapse-time window. The first data group (a) consists largely of the data of Herrmann (1980) and Singh & Herrmann (1983) for continental USA, and of Jin & Aki (1988) for China. These data show large scatter. Furthermore, of the seven cases in which Jin & Aki (1988) were able to determine Q_c in two consecutive lapse-time windows, in five cases the trend was opposite to the one expected here. In both studies, plotted Q_c estimates were based on the analysis of instant coda frequency on the record of a conventional seismograph. There is as yet no fully convincing explanation for the observed scatter.

The second data group (b) behaves better. Only three points show prominent (more than two times) deviations from the average trend: two lower ones for the Petatlan (1979, Mexico) earthquake aftershocks, and one high one for the Adak region (Aleutians). Therefore large deviations from the proposed relationship do exist. Taken as a whole, the spectrally analysed data shown in Fig 2 generally confirm our assumptions.

Data on the exponent β of eq. (15) are given in Fig. 3. For comparison, 'theoretical' curves are given, calculated in the following manner. n (=2.5) and Q_{int} (=2000 or 5000) are fixed, and, at each t, Q_t is calculated from eq. (13) for $f=1.5$ and 24 Hz. Then, $\beta = \beta(t) = d \log Q_t / d \log f$ is estimated based on these two frequency points. The curve for $Q_{int}=2000$ shows a qualitatively reasonable average trend but is systematically too low. Though this error can be greatly reduced with $Q_{int}=5000$, this value of Q_{int} seems too large. The overall fit is far from ideal; one suspects some additional factors at play. In particular, one might assume the n value to be frequency-dependent, with faster depth decay of turbidity at higher frequencies; with such a modification, the large β values (equal to or above unity) seen in the figure can be readily explained.

Fig. 4 shows limited observational material on coda shapes proper, again for 1.25–1.5 Hz. For comparison, theoretical

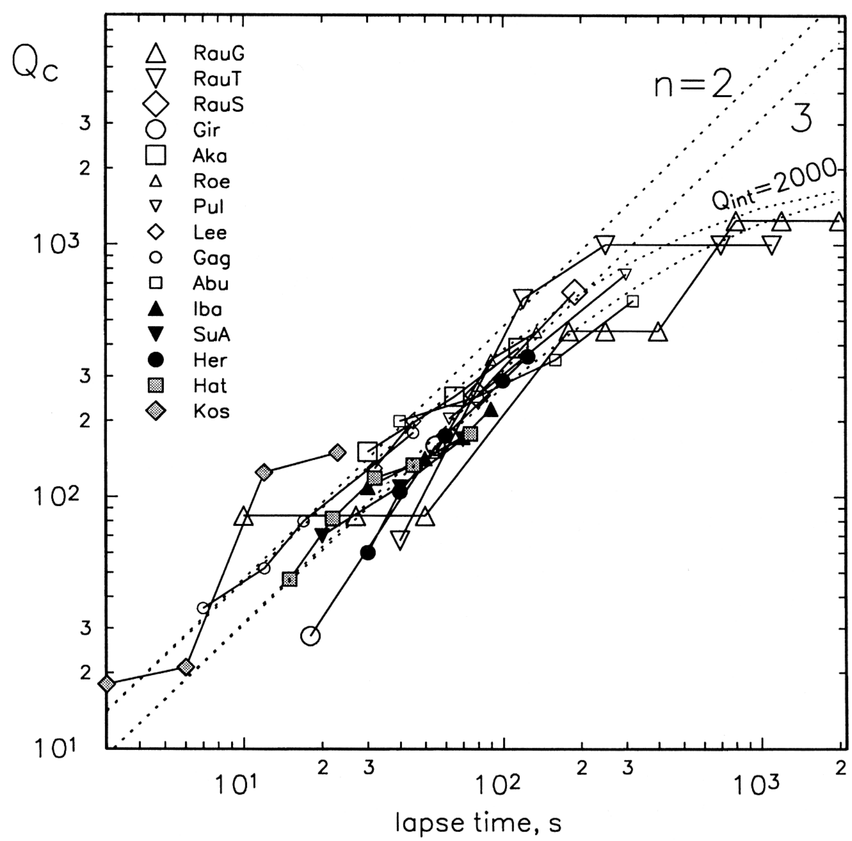

Figure 1. Published observed coda Q versus lapse time for $f=1.5$ Hz, for studies where spectral analysis was carried out and coda Q versus t variation was explicitly assumed during data interpretation. Source codes are given in Table 1. Dashed lines correspond to eqs (12) and (13) with parameters shown on the plot.

Table 1. Data sources.

Reference	code	t_1–t_2[#]	Comment[##]
			Figure 1
Rautian & Khalturin (1978)	RauG	10–2000	Garm station, Pamir.*
Rautian *et al.* (1981)	RauT	20–120	Talgar station, Tien-Shan.*
Rautian *et al.* (1981)	RauS	30–250	Shikotan station, S.Kuriles.*
Gir Subhash & Gir (1979)	Gir	8–65	Jura station, Germany
Akamasu (1980)	Aka	15–150	Sumiyama station, Japan
Roecker *et al.* (1982)	Roe	15–160	Data of Fig. 4**
Pulli (1984)	Pul	25–500	
Lee *et al.* (1986)	Lee	20–60	**
Gagnepain-Beyneix (1987)	Gag	4–90	Data of Fig. 5, averaged
Abubakirov & Gusev (1990)	Abu	32–400	Three stations, averaged
Ibañez *et al.* (1990)	Iba	20–130	
Su *et al.* (1991)	SuA	3–93	
Herak (1991)	Her	30–130	
Kosuga (1992)	Kos	2–30	
Hatzidimitriou (1993)	Hat	10–100	
			Figure 2
Herrmann (1980)	Herr		Station DUG§¶
Singh & Herrmann (1983)	Herr		Only plotted data from 1-s instruments§¶
Canas (1986)	Cana		Two subregions§¶
Canas *et al.* (1988)	Cana		§¶
Jin & Aki (1988)	JinA		Best data (A/α, B/α)§¶
Xie & Nuttli (1988)	Xie		Two stations averaged,¶
Xie & Mitchell (1990a)	Xie		Typical Q_c value over many stations¶
Xie & Mitchell (1990b)	Xie		¶
Tsujiura (1978)	Tsuj	25–125	Average over two stations, Japan
Rovelli (1982)	Rove	12–25	
Rovelli (1984)	Rove	13–20	
Rodriguez *et al.* (1983)	Rodr	40–110	
Del Pezzo *et al.* (1983)	DelP	10–40	
Del Pezzo & Zollo (1984)	DelP	7–30	
Del Pezzo *et al.* (1985)	DelP	5–45	
Rhea (1984)	Rhea	5–85	Body-wave coda interpretation questionable at 1 Hz
Scherbaum & Kisslinger (1985)	Sche	5–50	
Rebollar *et al.* (1985)	Rebo	6–12	**
Rebollar *et al.* (1987)	Rebo	20–40	**
Phillips *et al.* (1988)	Phil	15–30	Three regions averaged
Havskov *et al.* (1989)	Havs	18–38,	Oregon average
		10–20	Mt St Helens
Steensma & Biswas (1988)	Stee	30–120	
Van Eck (1988)	VanE	20–60	**
Kvamme & Havskov (1988)	Kvam	20–90	Average for 30 s and 40 s windows
Ambeh & Fairhead (1987)	Ambe	20–50	
Novelo-Casanova *et al.* (1990)	Nove	15–25	**
Hoshiba (1993)	Hosh	45–85	16 stations, averaged

#lapse-time range, explicitly stated or deduced from data description. Not given for unfiltered data.
##study region is mentioned only if absent in the title.
*part of the original Q_c values were based on the surface-wave model; these were converted to the body-wave model.
**extrapolated from 3 Hz.
§unfiltered records used.
¶original Q_c is plotted, based on the surface-wave model.

curves are plotted according to eqs (6) and (14) ($A_c \propto \sqrt{I_n}$). One can see reasonable overall agreement. It is even possible to hypothesize that n varies from region to region, with $n \approx 2$ in W. Pamir, $n < 2$ in Tien-Shan and $n \approx 2.5$–3 in the Kuriles and Kamchatka. Superposed on the general shape, some detailed variations are seen on the curves. These probably reflect individual deviations of particular turbidity profiles under each station from the assumed smooth profile. One can ask why these evident deviations of coda decay curves from a simple theoretical shape (eq. 14), as well as the variations of n, can produce only limited deviations of observed Q_c values from the theoretically expected trend (eq. 12), as seen in Figs 1 and 2.

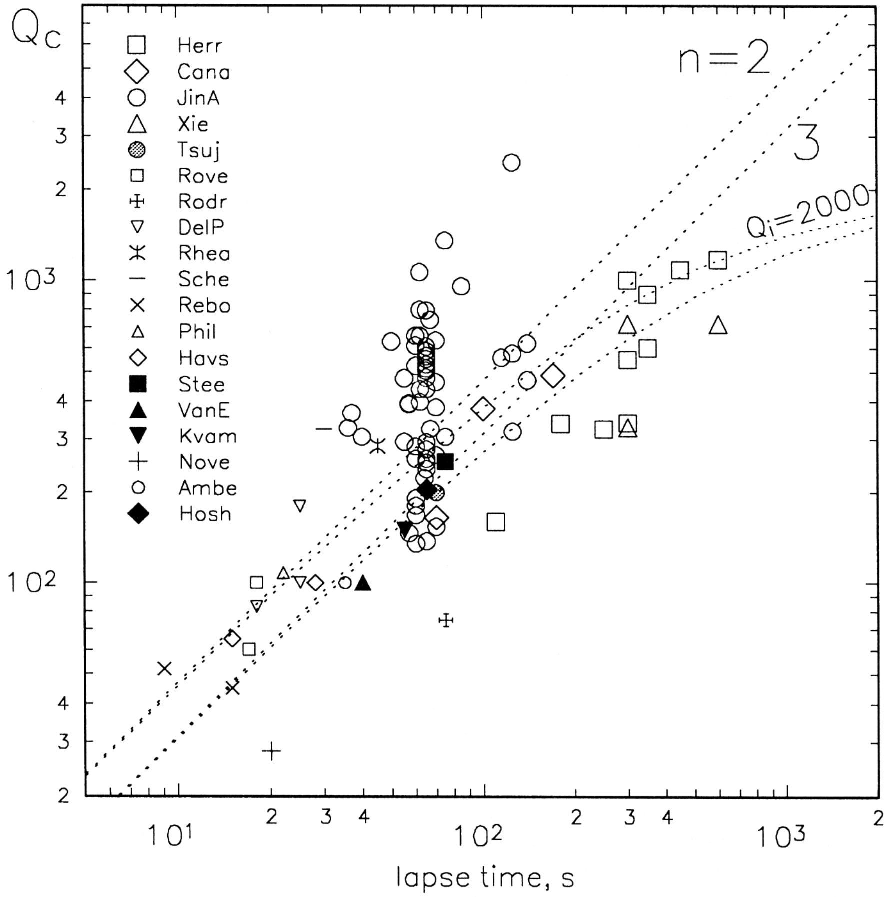

Figure 2. As Fig. 1, but for studies where either unfiltered records of conventional instruments were used, or where filtered data were analysed with no account of possible Q_c versus t variation.

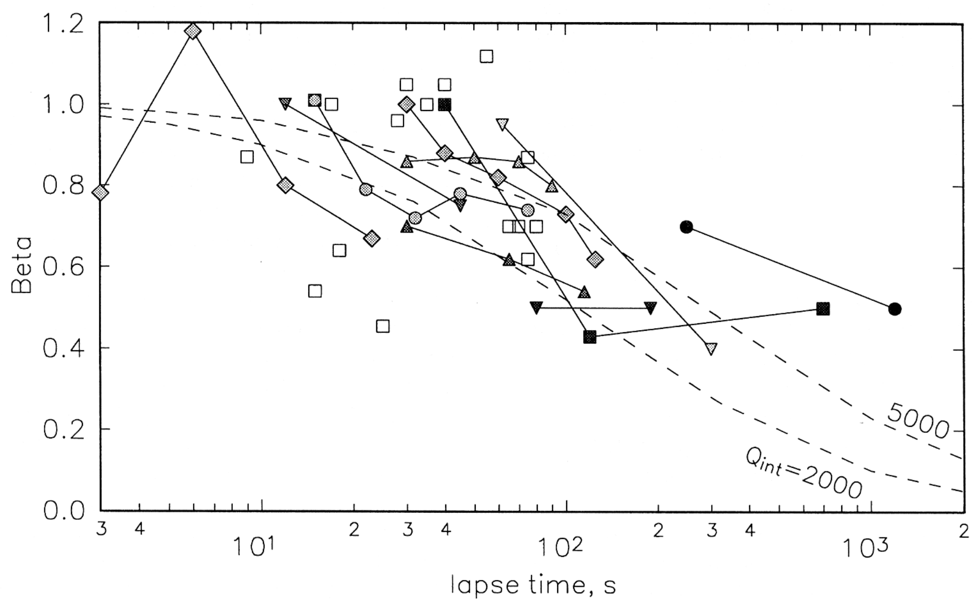

Figure 3. Published observed spectral exponent β of eq. (15) according to sources listed in Table 1. Dashed lines show the β versus t behaviour expected for the present model (see text for details).

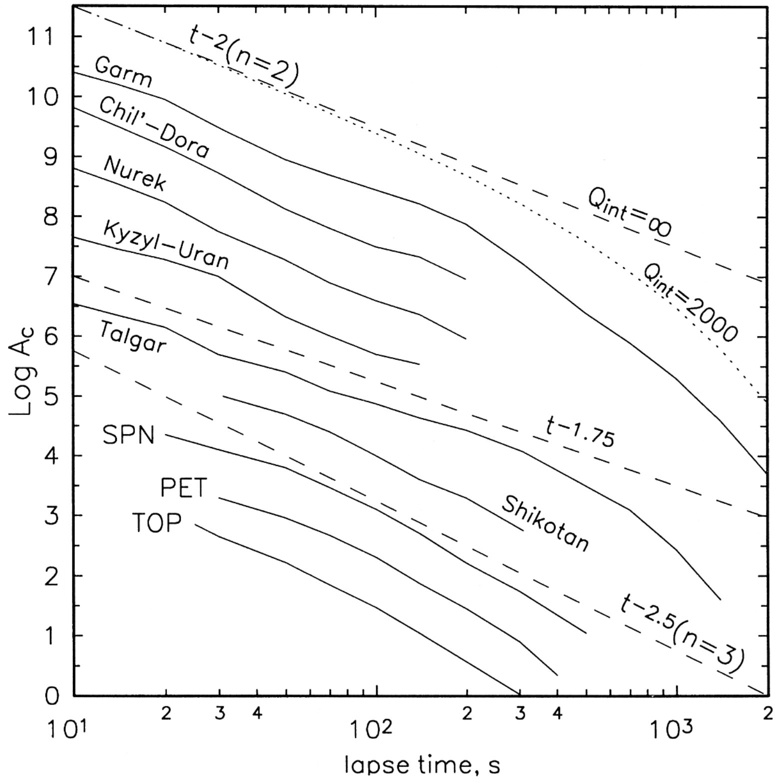

Figure 4. Observed average coda shapes. Results for stations Garm (W. Pamir), Talgar (N. Tien-Shan, $t = 100$–1200 s only) and Shikotan (S. Kuriles) are from Rautian *et al.* (1981). Results for stations Chil'-Dora (W. Pamir), Nurek (W. Pamir), Kyzyl-Uran (near Toktogul, Tien-Shan) and Talgar for 10–100 s are from Kopnichev (1985). The other three are Kamchatka stations; results are from Abubakirov & Gusev (1990). Central frequencies are 1.25, 1.4 or 1.5 Hz. Theoretical shapes (eqs 6 and 14) are shown for comparison.

This discrepancy is apparent, and is due primarily to the wide logarithmic span of Figs 1 and 2. Note also that in view of this span, the Q_c dependence on n (eq. 12) is weak.

The overall fit of spectrally analysed data by the simple model proposed here is rather good, and captures the gross features of the turbidity structure in the Earth. The value of the exponent n seems to be in the range 2–3, and Q_{int} is of the order of 2000. Deviations of the observed Q_c values from predictions of the model are usually no more than 1.5 times, and rarely exceed 2 times.

DISCUSSION

The conclusions of this paper are now compared with those of Rautian *et al.* (1981). In their model based on Garm station data, the turbidity g drops 40 times between the upper ($h = 0$–[10–45] km) and the second ($h =$ [10–45]–600 km) layer, and 5000 times more between the second layer and the rest of the mantle. Roughly, this means that g drops 20 000 times between $h = 10$ and $h = 1000$ km, pointing to $n \approx 2.2$, which is within the preferred interval $n = 2$–3.

One may ask whether ignoring multiple scattering and non-isotropy of scattering (as is done here) can be justified. A fully convincing answer can only be found by direct modelling, which is outside the scope of the present paper. However, a strong argument in favour of a single-scattering approximation for the Earth is the presence of energetic direct body-wave arrivals in most Earth seismograms—a fact which definitely indicates that propagation distance is typically small compared to the ray-averaged mean free path, or at least comparable to it.

It should be noted that the simplicity of eqs (6) and (7) and the absence of exponentially decaying terms of the form $\exp(-Ct)$ on the right-hand side of eq. (6) depends strictly on the condition $r \ll H$. In other words, our theory is valid for the case of a thin upper layer. Therefore one cannot expect the model to work at lapse times $t < t_{cr} = H/c$. Unfortunately, there are insufficient data to estimate H or t_{cr}. Furthermore, coda decay curves can differ markedly among individual seismic stations of a region, especially at small lapse times (Rautian *et al.* 1981), and thus one should not expect any universal H value. For sedimentary basin environments (Spudich & Iida 1993 and references therein), the upper few kilometres seem to contain the strongest scatterers; the role of topography ('zero-depth' scatterer) is also important. On the other hand, individual strong scatterers inverted by Nishigami (1991) occur over the entire 0–30 km depth range. Therefore, t_{cr} can be thought to be somewhere in the interval 2–8 s. The data in Fig. 1 do not show any clear manifestation of t_{cr}.

One can ask how the high intrinsic Q estimate of about 2000 compares with some much lower intrinsic Q estimates reported for the lithosphere (for example Gusev & Lemzikov 1985; Hoshiba 1993). These estimates from scattered waves are based on the assumption of a uniformly scattering medium; from the viewpoint of the present model they are considered to be outdated. However, our results compare well with those of Toksoz *et al.* (1988) and Hough *et al.* (1988) below the upper few kilometres of high attenuation. Rough estimates predict that the addition of a highly attenuative 'cap' will add a nearly constant factor to (14) and thus will not bias our results regarding coda Q.

Sato (1988) proposed a random fractal scatterer distribution which predicts power-law coda decay akin to the one proposed here. Despite certain conceptual differences (random uniform scatterers and spherical geometry of the calculation in Sato (1988), as compared with non-uniform scatterers and layered geometry here), his results are comparable to these. For his analytically more simple model, Sato found that for cases in which the single-scattered coda amplitude decays faster than t^{-1} (the case that covers the domain of $n=2$–4 studied here), the multiple-scattered energy will decay faster than single-scattered energy, thus becoming negligible at large t and justifying the use of SISM. Some similarity exists between Sato's model and this one, giving additional support for the neglect of multiple scattering.

It is worth mentioning that, in the present model, there is no relation between Q_c and any absolute measure of scattering, such as G or GH. (However, the absolute coda level depends on these.) This property of the model is somewhat unexpected, but can be understood if one takes into account the fact that coda Q reflects the decay rate of coda, whereas the turbidity value determines the absolute coda level. Moreover, the linear increase of loss-less coda Q with frequency, inherent in the present model, has nothing in common with the frequency behaviour of turbidity. This property of the present model is radically different from Dainty's (1981) model for Q_S, which, despite an almost identical functional form, is based on the assumption of frequency-independent turbidity; i.e. Dainty (1981) explicitly assumes a particular turbidity versus frequency relation (constant), in contrast to the present model where absolute turbidity values are not constrained at any frequency. This is a particular manifestation of the general fact that, in the present model, Q_S and coda Q are independent (whereas Q_S and g are closely related). The present model shares one element with the concepts of Aki (1980) and Dainty (1981): at around 1–2 Hz, and at low to moderate lapse times, coda Q (in common, presumably, with Q_S) has almost no relation to intrinsic loss. This model, however, provides no theoretical ground for the numerical agreement between Q_c and Q_S (Rautian & Khalturin 1978; Aki 1980; and later work, e.g. Rovelli 1982; Kvamme & Havskov 1988).

CONCLUSIONS

Reasonably good agreement is found between coda Q measurements in the lapse-time range 3–2000 s and frequency range 1–2 Hz, based on band-filtered or spectrally analysed data, and predictions of a vertically varying turbidity model with a truncated power-law profile of a rather steep nature (inverse square or inverse cube of depth), with a thin upper layer. The frequency dependence of coda Q qualitatively agrees with the model. These facts probably mean that the model captures the gross features of the real turbidity profile of the Earth.

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